Identification of a Cessna Citation X Aero-Propulsive Model in Climb Regime from Flight Tests

Georges Ghazi, Ruxandra M. Botez and Magdalena Tudor
École de Technologie Supérieure – University of Quebec
Laboratory of Applied Research in Active Controls, Avionics and AeroServoElasticity
Montreal, Quebec, Canada

http://www.larcase.etsmtl.ca / ruxandra.botez@etsmtl.ca

Abstract—During aircraft development, several mathematical models are created from our knowledge of fundamental physical laws. Those models are used in order to make decision at all development stages. In this paper, a methodology to design an aero-propulsive model for the Cessna Citation X in climb regime from flight test identification to model identification is presented. The aircraft’s model was built by identifying a general aircraft mathematical model in climbing flight. A professional level D flight simulator was used as a flight test aircraft and a total of 70 flight tests were performed at different flight points within the aircraft flight envelope. The obtained aero-propulsive model was next interpolated to provide a performance database model within the whole aircraft flight envelope. Results showed that the proposed methodology gives an excellent estimation of the aircraft performance with a success rate of 100% for both identification and validation process.

Keywords—Aero-Propulsive model; Cessna Citation X; Level D Flight Simulator; Flight Tests; System Identification

I. INTRODUCTION

Since 1980, the airplane has become the most common way to travel great distances. According to the Air Transport Action Group (ATAG) [1], in 2014, more than three billion passengers boarded an aircraft in order to travel somewhere on earth. Although this mobility has a beneficial impact on the global economy and international trade between countries [2], it partly contributes to global warming. According to the ATAG [3], in 2011, the airline operations were responsible of around 2% of carbon dioxide (CO₂) global emissions. Even if this percentage is still relatively low compared to other transports, the aerospace industry aims to reduce by 50% its carbon footprint in 2050 [1].

Over the last years, several researches and techniques have been elaborated in order to improve aircraft performances. According to Okamoto et al. in [4], a 20% reduction in airplane drag can reduce up to 18% on fuel consumption. Such drag reduction can be achieved through the implementation of winglets wingtip devices on current commercial aircraft [5, 6]. According to Boeing, this improvement led to increase the new Boeing 737 MAX fuel efficiency by 1.8%. Similarly, morphing wing technologies can be used to reduce the airplane drag, and thus reduce the fuel consumption. According to Gabor et al. in [7-9], a local modification of the aircraft wing shape could improve the aerodynamic characteristics of the wing in flight, and therefore reduce the airplane drag.

Improving the engine efficiency is also an ongoing effort of engine manufacturers. For example, the new Airbus 320 NEO (New Engine Option) has been equipped with new CFM International Engines more powerful and sharklets (winglets). According to Airbus [5], these improvements will reduce the fuel consumption by 15% compared to a conventional Airbus A320. Biofuel also is a very promising alternative to reduce CO₂ emissions [10].

All these examples highlight the efforts provided by the aerospace industry to reduce its overall carbon footprint. However, although these techniques are promising, they cannot be implemented on aircrafts that are currently in service. It is therefore of interest to find other alternatives. According to Jesen et al. in [11, 12], most of aircraft in the United State do not flight at their optimal trajectories. This is the reason why, these last years, the aerospace industry and several researchers have focused their studies on trajectories optimization [13-19]. By reducing both flight distance and flight time, trajectories optimization leads to reduce the fuel consumption, and so the CO₂ emissions.

Trajectory optimization in vertical or lateral profile is the main function of the Flight Management System (FMS) [20]. The FMS is an airborne device used by the pilot or the airline to predict the optimal trajectory that minimizes the flight cost expressed in terms of flight time and total fuel burned. To estimate the aircraft performances and compute the optimal trajectory, the FMS needs a mathematical representation of the aircraft [21, 22]. Such a representation can be obtained from a set of nonlinear equations also called Equations of Motion (EoM). For example, Ghazi and Botez in [23, 24] presented a full nonlinear model of the Cessna Citation X business aircraft that can be used to estimate and analyze the aircraft performance for any flight phase. However, because of limited processing capacity, a FMS cannot support an aircraft model based on EoM. It is therefore of interest to build another aircraft model, which is more adapted to the FMS’ architecture.

Sibin et al. in [25] described a methodology to develop an aircraft performance model for the flight management system using data obtained from a flight simulator prototype. The obtained model provided good performance data such as aerodynamic forces or engine thrust, and can be therefore used to describe the aircraft behavior within its flight envelope. However, as some aircraft manufacturers are conservative to provide complete aero-propulsive data, having access to the
Aircraft data required to create such a model can be very difficult.

Murrieta et al. in [26] presented a methodology to create an aircraft Performance Database (PDB) using a Citation X Level D flight simulator. Based on several flight tests, the aircraft fuel flow was sampled during the cruise phase for different constant altitudes and speeds. The aircraft mass was also considered constant. The results were next prepared and formatted into lookup table in order to be used by an in-house algorithm that can predict the fuel burn during cruise. However, the methodology proposed by Murrieta et al. did not provide enough information about the aircraft aerodynamic parameters, which are usually necessary for aircraft performance analysis or for flight control system design purpose.

The main objective of this paper is to present a methodology for deriving an aero-propulsive model of the Cessna Citation X from flight tests that will allow to have a better estimation of the aircraft performance in climb regime. Such a model could be useful to support the researchers in order to validate their algorithms for trajectory optimization [21, 22, 27] and/or flight control system [28-31]. The flight tests were performed on a professional Cessna Citation X level D aircraft research flight simulator (see Fig. 1) designed and manufactured by CAE Inc. According to the Federal Aviation Administration (FAA, AC 120-40B), the level D is the highest certification level for the flight dynamics modeling.

This paper is organized as follows. In Section 2, the aero-propulsive performance model structure and the problem statement are presented. Section 3 deals with the methodology used to build the aero-propulsive model. Section 4 presents the results of a case study in which the methodology was applied to predict the aircraft performance of the Cessna Citation X during climb. Finally, the paper ends with conclusions and future work remarks.

II. AERO-PROPELLIVE MODEL AND CLIMB TRAJECTORY PREDICTION

This section first introduces a description of an aero-propulsive model. Then, after a presentation of the different mathematical equations that define the aero-propulsive model, the algorithm used to predict the aircraft trajectory is shown.

A. Aero-Propulsive Mathematical Model

By definition, an aero-propulsive model is used to predict the force acting on an aircraft under specific flight conditions and for a given aircraft configuration. In general, an aero-propulsive model consists of two sub-models: one sub-model is used to predict the aerodynamic drag force, while the other is used to estimate the propulsive thrust force.

As illustrated in Fig. 2, an aero-propulsive model can be compared to a black box with multiple inputs and outputs. The choice of these inputs and outputs depends mainly on the study of interest. As shown in many studies [17, 26], the main parameters that affect the aircraft behavior in climb regime are the gross weight, the center of gravity position, the altitude, the speed and the ISA (International Standard Atmosphere) temperature deviation, while the outputs of interest are usually the drag and engine thrust forces. Therefore, an aero-propulsive model can be represented in a more mathematical form by the following general equation:

\[ [D, T, T_{sfc}] = f(GW, X_{cg}, IAS, h, \Delta ISA) \]  \hspace{1cm} (1)

where \( D \) is the drag force, \( T \) is the engine thrust force, \( T_{sfc} \) is the engine specific fuel consumption, \( GW \) is the gross weight, \( X_{cg} \) is the center of gravity position, \( IAS \) is the indicated airspeed, \( h \) is the altitude, \( \Delta ISA \) is the temperature deviation and \( f: \mathbb{R}^5 \rightarrow \mathbb{R}^3 \) is the mathematical representation of the aircraft performance (i.e. the aero-propulsive model).

1) Aircraft Equations of Motion in Climbing Flight

The development of the aero-propulsive mathematical model starts with the kinetic equations of motion. Based on the Newton’s second law, namely

\[ \frac{W}{g} a = \Sigma F \]  \hspace{1cm} (2)

where \( \Sigma F \) is net force applied to the aircraft and \( a \) is the aircraft acceleration relative to the inertial frame, the kinetic equations of motion for an aircraft in climbing flight can be written in the stability axes as follows:

\[ \frac{W}{g} \frac{dV}{dt} = T - D - W \sin(\gamma) \]  \hspace{1cm} (3)

\[ \frac{W}{g} \frac{d\gamma}{dt} = -L + W \cos(\gamma) \]  \hspace{1cm} (4)

where, \( g \) is the acceleration due to gravity, \( V \) is the aircraft true airspeed and \( \gamma \) is the aircraft climb path angle.
Equations (3) and (4) have the advantage to describe the complete aircraft longitudinal motion. According to these equations, if the forces acting on the aircraft (i.e., lift, drag, and thrust) are known, therefore the aircraft trajectory can be estimated. Conversely, if the aircraft trajectory is known, therefore the forces can be determined. In others words, using sampled data from flight tests for different flight conditions and aircraft configurations, a model for the forces can be identified. Then, using this same model, the aircraft trajectory in climb can then be predicted for any flight condition and aircraft configuration within the aircraft flight envelope.

2) Lift and Drag Forces Estimation

According to several references in aircraft flight mechanics [32-36], the two components of the aerodynamic forces $L$ and $D$ can be expressed with non-dimensional coefficients $C_L$ and $C_D$ such as:

$$D = qSC_D$$  \hspace{1cm} (5)

$$L = qSC_L$$  \hspace{1cm} (6)

where $q = 1/2\rho V^2$ is the dynamic pressure and $S$ is the reference wing area. The lift force $L$ can be easily obtained from Eq. (4) as follows:

$$L = W \cos(\gamma) - \frac{W}{g} \left[\frac{d\gamma}{dt}\right] V$$ \hspace{1cm} (7)

Then, by combining Eq. (11) and Eq. (12), the lift coefficient $C_L$ can be determined with the next equation:

$$C_L = \frac{1}{qS} \left( W \cos(\gamma) - \frac{W}{g} \left[\frac{d\gamma}{dt}\right] V \right)$$ \hspace{1cm} (8)

Finally, based on the result in Eq. (8), the drag aerodynamic coefficient can be therefore estimated from the drag polar equation of a cambered wing [33, 35, 36], which states that:

$$C_D = C_{D_{min}} + \frac{C_L^2}{\pi AR e \sqrt{1 - M^2}}$$ \hspace{1cm} (9)

where $C_{D_{min}}$ is the minimum drag coefficient, $AR$ is the aircraft aspect ratio, $e$ is the Oswald efficiency factor and $M$ is the aircraft Mach number.

3) Engine Thrust and Specific Fuel Consumption Estimation

Using the estimation of the drag force obtained in the previous section, the thrust force can be therefore determined from Eq. (3) such as:

$$T = \frac{W}{g} \left[\frac{dV}{dt}\right] + D + W \sin(\gamma)$$ \hspace{1cm} (10)

Then, the engine specific fuel consumption coefficient $T_{sfc}$ can be estimated from Eq. (11),

$$T_{sfc} = \frac{W_f}{T}$$ \hspace{1cm} (11)

where $W_f$ is the engine fuel flow defined by:

$$W_f = \frac{dFB}{dt}$$ \hspace{1cm} (12)

and $FB$ is the fuel burn.

This last equation concludes the aircraft aero-propulsive mathematical model. In the next section, the algorithm used to predict the aircraft climb trajectory is presented.

B. Climb Trajectory Prediction

To predict the aircraft trajectory, two parameters must be computed: the altitude and the horizontal distance. However, both parameters depend mainly on the rate of climb. Thus, it is first necessary to find a way to express the rate of climb.

Equation (3) is rearranged as follows:

$$\frac{T - D}{W} = \sin(\gamma) + \frac{1}{g} \frac{dV}{dt}$$ \hspace{1cm} (13)

Then by noticing that:

$$\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} \quad \text{and} \quad \frac{dh}{dt} = V \sin(\gamma)$$ \hspace{1cm} (14)

and by replacing Eq. (14) into Eq. (13), the following Eq. (15) is obtained:

$$\frac{T - D}{W} = \left(1 + \frac{V}{g} \frac{dV}{dh}\right) \frac{h}{V}$$ \hspace{1cm} (15)

Finally, from Eq. (15), the rate of climb $\dot{h}$ can be expressed as follows:

$$\dot{h} = \frac{T - D}{W} \frac{V}{(1 + AF)}$$ \hspace{1cm} (16)

where $AF$ is the acceleration factor defined by:

$$AF = \frac{V}{g} \frac{dV}{dh}$$ \hspace{1cm} (17)

The altitude $h$ can be therefore obtained by integrating the result in Eq. (16). However, to performed numerical integration, the aircraft trajectory should be divided into $N-1$ sub-segments separated by $\Delta h = 1,000$ ft as shown in Fig. 3. Thus, for each sub-segment $\Delta h$, the drag and thrust forces are first computed using the identified aero-propulsive model. Then, based on these estimations, the average rate of climb for each sub-segment is estimated. Finally, the flight path angle $\gamma$ is computed using the Eq. (14).

In parallel, the engine fuel flow is also estimated using the engine specific fuel consumption parameter (see Eq. (18)).
Equation (18) shows the complete procedure to estimate all the aircraft parameters for a given sub-segment,

$$\begin{aligned} h_i &= \frac{T_i - D_i}{W_i} V_i \\
\gamma_i &= \text{asinh} \left( \frac{h_i - h_{i+1}}{V_i} \right) \\
\bar{G}_S_i &= V_i \cos(\gamma_i) \\
\bar{W}_i &= T_i \times T_{sfc_i} 
\end{aligned}$$  \tag{18}

where $\bar{G}_S_i$ is the average ground speed for a sub-segment defined by the altitudes $h_i$ and $h_{i+1}$, $i \in [1, N]$.

Finally, the aircraft horizontal distance $HD$ traveled and the fuel burn $FB$ were determined using an Euler integration method as follows:

$$\begin{aligned} HD_{i+1} &= HD_i + \bar{G}_S_i \Delta t_i \\
FB_{i+1} &= FB_i + \bar{W}_i \Delta t_i 
\end{aligned}$$  \tag{19, 20}

where $\Delta t_i$ is the time to climb between two consecutive altitudes.

III. SYSTEM IDENTIFICATION AND PARAMETER ESTIMATION ALGORITHM

According to Zadeh [37], “system identification is the determination, on the basis of observation of input and output, of a system within a specified class of system to which the system under test is equivalent”. As shown in Fig. 4, system identification includes the model structure definition based on mathematical equations and the estimation of parameters defining the model.

Based on these observations, the proposed methodology to identify an aero-propulsive model from flight tests consists of two steps. In a first step, several flight tests have to be performed in order to sample the inputs and outputs required to describe the aircraft performance. Then, in a second step, a procedure that automatically tunes the parameters defining the model according to the study in Section II must be developed.
TABLE I. SAMPLE CLIMB DATA

<table>
<thead>
<tr>
<th>Altitude (ft)</th>
<th>Fuel Burn (lbs)</th>
<th>Horizontal Distance (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2,000</td>
<td>25.55</td>
<td>2.29</td>
</tr>
<tr>
<td>3,000</td>
<td>50.76</td>
<td>4.61</td>
</tr>
<tr>
<td>33,000</td>
<td>722.44</td>
<td>96.49</td>
</tr>
<tr>
<td>34,000</td>
<td>743.80</td>
<td>100.27</td>
</tr>
<tr>
<td>35,000</td>
<td>765.42</td>
<td>104.12</td>
</tr>
</tbody>
</table>

B. Parameter Estimation Algorithm

The aero-propulsive model is derived by determining a combination of thrust and drag forces that best estimates the rate of climb. Thus, for each sub-segment, the climb path angle, the rate of climb, the time to climb and the engine fuel flow were computed using Eqs. (21) to (24):

\[
\gamma_i = \arctan\left(\frac{h_{i+1} - h_i}{d_{i+1} - d_i}\right) \quad (21)
\]

\[
\hat{h}_i = \bar{V} \sin(\gamma_i) \quad (22)
\]

\[
\Delta t_i = \frac{h_{i+1} - h_i}{\hat{h}_i} \quad (23)
\]

\[
W_{f_i} = \frac{FB_{i+1} - FB_i}{\Delta t_i} \quad (24)
\]

where \(\bar{V}\) is the average true airspeed along a sub-segment, \(\Delta t_i\) is the time to climb from \(h_i\) to \(h_{i+1}\) and \(W_{f_i}\) is the average engine fuel flow along the sub-segment. In the same way, the acceleration factor \(AF\) and the Mach number were also determined along each sub-segment. As all these values are computed directly using the sampled data in Table 1, they are assumed to represent the real state of the aircraft.

In parallel, using a first initialization for the minimum drag coefficient \(CD_{\text{min}}\) and the Oswald efficiency factor \(e\), the drag and the thrust forces were calculated from Eqs. (5), (9) and (10). These results allow to find a first estimation of the rate of climb using Eq. (16). A minimization routine based on the Nelder-Mead algorithm [38] was next used to adjust the minimum drag coefficient, the Oswald efficiency factor and the thrust in order to minimize the error between the estimated rate of climb obtained with Eq. (16) and the rate of climb computed with Eq. (22). Then, the engine specific fuel consumption coefficient \(T_{sfc}\) was computed using the optimal thrust and rate of climb resulting from the minimization, and Eqs. (11) and (12) such as:

\[
T_{sfc} = \frac{\hat{h} \Delta FB}{T \Delta h} \quad (25)
\]

where \(\Delta FB\) is the difference of fuel burn between two altitudes of a sub-segment.

The complete procedure of the estimation algorithm applied for one flight test (i.e. for one aircraft configuration) is shown in Fig. 6.
The procedure shown in Fig. 6 was applied on 9 of the 70 flight tests performed with the level D simulator. For each flight test, the drag force $D$, the thrust force $T$, and the thrust specific fuel consumption coefficient $T_{sfc}$ resulting from the minimization routine were stored and formatted into different 3-D lookup tables as shown in Fig. 7.

Fig. 7. 3-D Lookup Table Illustration

or in a more mathematically form as follows:

$$
[D] = f_{DP}(GW, IAS, h) \\
[T] = f_{F}(GW, IAS, h) \\
[T_{sfc}] = f_{sfc}(GW, IAS, h)
$$

(26)

Finally, using a 3D linear interpolation, the three parameters defining the aero-propulsive model in Eq. (26) were interpolated in order to predict the aircraft trajectory for all the remaining 61 flight tests according to the procedure described in section Climb Trajectory Prediction.

IV. RESULTS

To validate the aero-propulsive model developed in this paper, 70 flight tests were performed using the level D Cessna Citation X flight simulator. Then, as mentioned in the section Flight Tests Description, these flights were divided into two categories. Only 9 flight tests were selected for the identification process, while the remaining 61 flight tests were used to validate the obtained model within the Cessna Citation X flight envelope (see Table 2).

To conclude about the efficiency of the proposed methodology, each flight test was compared against the Level D flight simulator. To do that, the horizontal distance traveled and the fuel burn were first computed from data measured with the flight simulator. In parallel, the same flight test was evaluated using the aero-propulsive model and the procedure described in section Aircraft Trajectory Prediction. The fuel burn and the horizontal distance traveled were next compared in order to conclude about the accuracy of the model. If the maximum error between the two models was less than 5%, then the flight test was considered as successfully identified or estimated. To illustrate the way in which each flight test was validated against experimental data, an example of three successful cases is given in Figures 8 and 9.

<table>
<thead>
<tr>
<th>TABLE II. FLIGHT TESTS ENVELOPE LIMITS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Altitude</td>
</tr>
<tr>
<td>Speed (IAS)</td>
</tr>
<tr>
<td>Gross Weight</td>
</tr>
<tr>
<td>Xcg</td>
</tr>
</tbody>
</table>

Fig. 8. Altitude and Horizontal Distance Estimations

Figure 8-(a) shows three comparisons between the aircraft vertical trajectory measured with the flight simulator and the aircraft trajectory estimated with the model. Fig. 8-(b) exposes the relative error for each trajectory. A positive error means that the model was climbing slower than the flight simulator. It is clear that the aero-propulsive model was able to find a solution that fits the experimental data. Indeed, as it can be seen, the error is at least equal to 0.35%. Moreover, it should be noticed that the error decrease considerably with the altitude. This can be explained by the fact that the more the aircraft climbs, the longer the travelled distance is. Therefore, the model error becomes neglected and the relative error decreases. Thus, as longer the aircrafts travels, the more accurate the model is.

In a general way, as shown in Fig. 9-(a), same observations can be made for the fuel burn estimation. As shown in Fig. 9-(b), the relative error between the experimental data and the predicted fuel burn is always less than 5%.

Fig. 9. Fuel Burn Estimation
The same analysis was repeated for all the 70 flight tests in order to validate the accuracy of the aero-propulsive model within the entire aircraft flight envelope. Table 3 shows the success ratio obtained and the number of flight test realized for both identification and validation processes. As it can be seen, the methodology gives an excellent estimation of the aircraft performance. Indeed, all the criteria imposed in this paper are satisfied with a success rate of 100% for each flight test category (identification and validation).

V. CONCLUSIONS AND FUTURES WORKS

In this paper, an aero-propulsive model for the Cessna Citation X in climbing flight was created using identification techniques from flight tests. A total of 70 flight tests were performed with a professional level D flight simulator designed and manufactured by CAE Inc., where the level D is the highest certification level for the flight dynamics modeling.

A complete mathematical model of the aircraft in climbing flight was presented, and an estimation algorithm was developed to identify the different parameters of the model. The identified parameters that compose the aero-propulsive model were next formatted into 3-D lookup tables in order to allow their interpolation within the whole Cessna Citation X flight envelope.

Results showed that the proposed methodology gave an excellent estimation of the aircraft performance with a success rate of 100% for both identification and validation process. Thus, it has been concluded that the aero-propulsive model created in this paper were experimentally validated.

ACKNOWLEDGMENT

This work was performed at the Laboratory of Applied Research in Active Controls, Avionics and AeroServo-Elasticity research (LARCASE). The Aircraft Research Flight Simulator was obtained by Dr Ruxandra Botez, Full Professor, thanks to the research grants that were approved by the Canadian Foundation of Innovation (CFI) and Ministère du Développement Économique, de l'Innovation et de l'Exportation (MDEIE) and the contribution of CAE Inc. Thanks are due to CAE Inc. team and its leader Mr Ken Dustin, and to Mr. Oscar Carranza Moyao for their support in the development of the Aircraft Research Flight Simulator at the LARCASE laboratory. Special thanks are also due to Mrs Odette Lacasse at ETS for her support in the research proposal. Thanks are due also to the CMC Electronics-Esterline team, more specifically to Mr Reza Neshat and Mr Oussama Abdul-baki for their interest in this subject.

REFERENCES


[26] A. Murrieta-Mendoza, S. Demange, F. George, and R. Botez, "Performance DataBase creation using a level D simulator for Cessna Citation X aircraft in cruise regime."


[29] G. Ghazi and R. M. Botez, "Lateral Controller Design for the Cessna Citation X with Handling Qualities and Robustness Requirements," presented at the 62nd CASI Aeronautics Conference and AGM, Montreal, Quebec, Canada, 2015.


[31] Y. Bougari, R. M. Botez, F. Theel, and G. Ghazi, "Optimal flight control on Cessna Citation X aircraft using differential evolution," presented at the 33nd IASTED International Conference on Modeling, Identification and Control (MIC), Innsbruck, Austria.


