Aircraft Lateral Flight Optimization Using Artificial Bees Colony

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***Abstract*— Fuel powered flights release polluting emissions to the atmosphere. The aeronautical industry has set itself the goal of reducing their global emissions share. Flight trajectory optimization is a way to reduce fuel consumption, thus reducing fuel emissions. Wind has a strong influence in fuel consumption. Tailwinds are desirable since they "push" the aircraft to its destination incrementing the ground speed for a given true air speed. This paper presents an algorithm that implements the artificial bee colony metaheuristic optimization algorithm to find the combination of waypoints that reduce the flight time between the departure and the destination points. The trajectory analyzed is at fixed Mach number and a fixed altitude. Fuel burn is computed using a performance database developed using experimental test data. The algorithm has the peculiarity that it does not require a fixed grid to generate the generated trajectories. Results have shown that for all tests the algorithm is able to identify trajectories with favorable wind, reducing the flight time, thus the fuel consumption and the flight cost.**

Keywords—Trajectory; Optimization; ABC; Bee; Lateral, Navigation, Aircraft

I. INTRODUCTION

The aeronautical industry is responsible of 2% of the total dioxide carbon (CO_2) released to the atmosphere. For this reason the industry has set the goal of reducing the polluting emissions generated primarily by fossil fuel such as reducing by the year 2050 the CO_2 emissions to 50% of those recorded in the year 2005. [1, 2]

To reach the reductions emission goal, aircraft operations have been seen as an interesting way to reduce fuel consumption, thus fuel emissions. Airlines have implemented different methods to reduce flight consumption such as engine washing, reduction of Auxiliary Power Unit (APU), weight reduction among others [3]. The descent phase has been explored principally because during the approach and landing phase, the aircraft and its pollution (emissions and noise) are released near populated areas. The most important improvement during descent is the Continuous Descent Approach (CDA) [4, 5]. This approach, contrary to the typical descent, consists in setting the engines in IDLE consuming low quantities of fuel and descending at a constant angle (typical 3°). It is important to correctly execute the descent approach and landing since executing the missed approach procedure significantly augments the total flight costs [6, 7].

According to [8, 9], not all aircraft fly at their optimal speeds and altitudes. For this reasons, different algorithms have been implemented to reduce flight costs by providing the optimal flight conditions. Optimal Control has been used to solve the equations of motion as in [10-15]. Genetic Algorithms have been used to optimize the lateral navigation (LNAV) and/or the vertical Navigation (VNAV) reference trajectory [16-20]. Branch and Cut has been implemented to optimize the vertical reference profile [21, 22]. Dynamic programming has been used to optimize the VNAV [23] and the LNAV [24]. Dijkstra's algorithm was explored for commercial and general purpose aircraft [25, 26]. Techniques to reduce the search space to reduce the computation time were developed in [27, 28]. Most of these algorithms required a grid where the aircraft was imposed to flight from/to the available waypoints.

The objective of this paper is to develop an algorithm to find the set of waypoints (longitude, latitude) from the departure waypoint, generally the Top of Climb (ToC), to the destination waypoint, the Top of Descent (ToD) that minimizes the flight cost.

The desirable LNAV reference route is the one that minimizes the flight cost. This reference trajectory has certain weather conditions that decrease the required energy (and flight time) for the plane to link the arrival to the destination point. The desirable weather conditions consist mostly in a tailwind and low temperatures. To find these weather conditions, the Artificial Bee Colony (ABC) algorithm was implemented in this paper. The ABC algorithm identified the best weather parameters to reduce the flight cost instead of focusing only in fuel consumption. When a complete trajectory taking into account the weather only was identified (all waypoints from ToC to ToD), the flight cost taking into account fuel consumptions was evaluated

As the ABC algorithm presents a well balance between the search space exploration and the exploitation ability, it is a good choice for the aircraft LNAV optimization. The ABC nature of researching multiple trajectories at the "same time" (or iteration) allows avoiding the trajectory to stagnate at a local optimal. The constant optimization provided by the ABC provides a very fine flight plan, able to maximize the wind influence to decrease the fuel consumption. The flight considered was at fixed altitude and Mach number.

The paper is arranged as follows, the methodology used to compute the flight cost is described, followed by the search space definition, and then the ABC theory and its implementations are described followed by results, conclusion, and future work.

II. METHODOLOGY

A. *Flight Cost – The Performance Database*

The aircraft model used in this paper to compute the flight cost is given in the form of a database. This database was created using in-flight experimental performance data. This database was constructed and developed by our industrial partner and it is called a Performance Database (PDB). The PDB contains different flight phases such as climb, acceleration, cruise, deceleration, and descent. However, because this paper focuses in cruise at a fixed flight level, only the PDB cruise phase was used. A methodology to create a PDB was described in[29].

The PDB can be considered as a black box which receives inputs to provide the pre-defined outputs. For the cruise phase, the required inputs are: the aircraft weight (kg), the speed (Mach number), the altitude (ft.), and the international standard atmosphere temperature deviation (°C). As an output the PDB provides the fuel flow (kg/hr). All inputs must be provided in order to obtain the desired output.

The PDB being a database contain discrete input data. Some inputs in the PDB, especially the aircraft weight and temperature, do not contain all exact possible values required to compute the fuel burned. When the exact variable input provided to the PDB is not available, interpolations among the PDB input limits are executed to obtain the desired output for the required input parameters. These interpolations are performed normally for aircraft weight as fuel is being burned and standard temperature deviation as the aircraft moves through the atmosphere. A complete methodology to compute a flight cost using a PDB was presented in [30, 31].The function used to compute the flight cost is shown in (1).

$$FlightCost = FlightTime\left(FuelFlow + CostIndex\right) \qquad (1)$$

The Cost Index is a variable that translates the time cost in terms of fuel; it is defined by the airline and remains constant through the flight. It is the goal of the ABC algorithm to minimize (1).

B. *The Optimization Algorithm*

1) *The search Space*

The aircraft moves in environment, which will be referred as from now on as the “search space”. The search space is located within the atmosphere and evolves constantly, changing wind conditions, temperature, and pressure.

The algorithm developed in this paper did not require a fixed waypoints grid such as many of the algorithms in the literature. The imposed waypoints grids in the literature were fixed equidistant waypoint located at a given perpendicular distance from the geodesic (shortest path between two points in a sphere) reference waypoints. The search space proposed in this paper was composed of geodesic reference waypoints as well where the position of the grid waypoints can be located at any distance from the reference geodesic waypoint. The distances, perpendicular to the reference trajectory, are determined by the ABC algorithm in a dynamic way. The grid difference is shown in Fig. 1.

Some trajectories created with the dynamic grid are shown in Fig. 2, notice again that the created waypoints are parallel to the geodesic reference route created. The black centerline represents the waypoints that form the original flight plan, which for this study is the geodesic line. The red lines represent the search space limits. No waypoint will be outside of these borders. Waypoints that form alternate trajectories are placed perpendicularly from the original flight-plan waypoints. Notice how the distance between the created waypoints are at different distances from the geodesic are not imposed, not at equidistant multiple distances as in Fig. 1. The created waypoints are shown in blue. In other words, the grid waypoints are dynamic, and it is the algorithm that decides where to place them as long as they are perpendicular to the reference waypoint.

2) *The Artificial Bee Colony Background*

Artificial Bee Colony (ABC) is one of the most metaheuristic algorithms. This algorithm mimics the honey bees’ intelligent behavior their search for food sources. A set of honey bees make a swarm able to successfully accomplish tasks through social cooperation, the ABC was first defined in [32].

There are three types of bees in the ABC algorithm: the “employed bees”, the “onlooker bees”, and the “scout bees”. The employed bees search different food sources around the current food source in their memories. They share the known found food sources information to the onlooker bees. The onlooker bees tend to select the highest quality food sources from the ones found by the employed bees. A scout bee is an employed bee searching for a new food source.

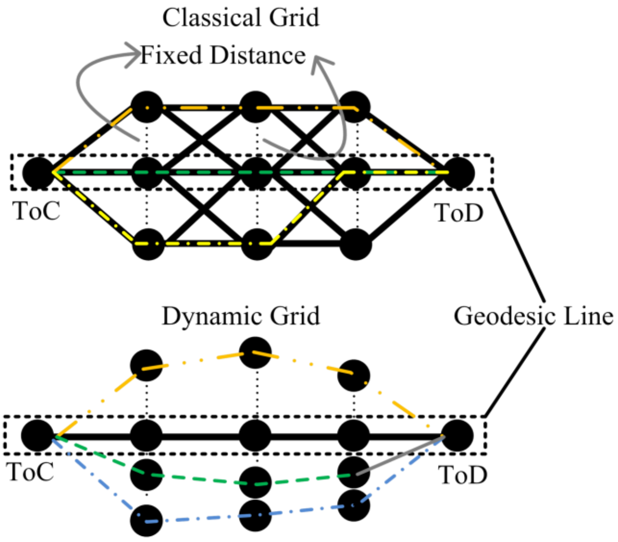

Fig. 1. Typical vs Dynamic Grid

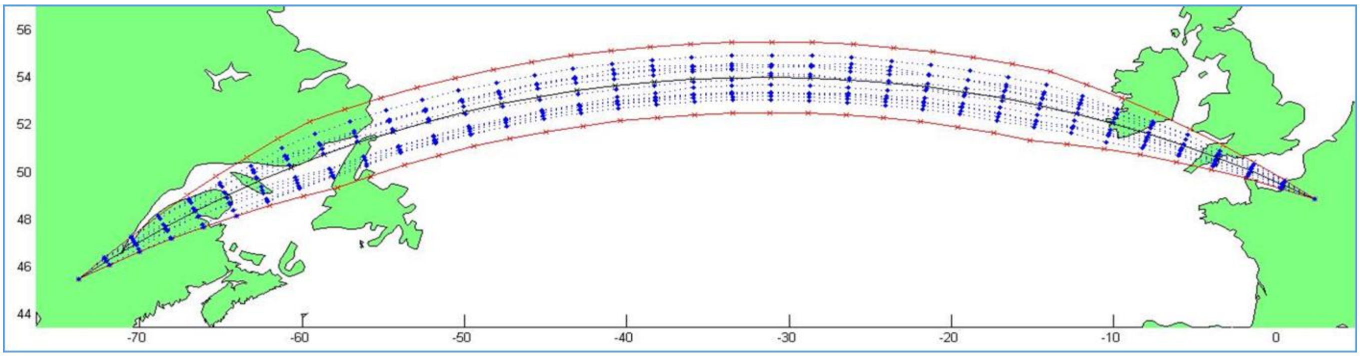

Fig. 2. Trajectories created using the dynamic grid.

3) *The ABC in the trajectory optimization*

The solution for the shortest LNAV reference trajectory problem can be defined as the set of waypoints from the ToC to ToD that minimizes the flight cost. For the ABC implementation, a given complete trajectory from ToC to ToD is assigned to a “bee” (it can be either employed, onlooker, or scout). The ABC algorithm methodology used in this paper is as follows.

Inputs: Departure and arrival Waypoint (Normally the ToC and the ToD). Weather information. Cost Index. Number of employed bees (*Ne*). Number of onlooker bees (*No*). Number of iterations, maximal counter number.

Phase I - Initialization: A predefined number of trajectories are created. The more trajectories there are; the more search space will be initially covered, but the computation time would increase. A fully-random trajectory creation would be time consuming. In order to reduce the algorithm execution time, there are pre-defined patterns in trajectory randomized creation. Every semi-random trajectory generated is assigned to an “employed bee”. Thus, there are an equal number of generated trajectories as there are employed bees. Trajectories are evaluated, by using its positions and the weather at the estimated passage time of the plane.

Phase II - Employed bee: In this step, every employed bee will create a random mutation on their assigned trajectory. The mutation is created by taking into account the behavior of the other trajectories. The more different trajectories there are the more variations in the mutations there will be. The mutated trajectory cost is evaluated and compared to the original trajectory. If the mutated trajectory is more economical than the original trajectory, it becomes the new trajectory for that employed bee; if the mutated trajectory is less economical than the current trajectory assigned the bee, it is discarded.

Phase III - Onlooker bee phase: Depending on their fitness, every trajectory explored by the employed bees is rated. The onlooker bees can follow any trajectory; however, they will be influenced by the trajectories rating. The higher a given trajectory rating, the more likely it is to be selected by several onlooker bees. As there are as many onlooker bees as employed bees, the trajectories with the lowest ratings may not be selected.

Every onlooker bee phase will mutate its selected trajectory, in the same way as during the employed bee phase. The mutated trajectory fitness is computed. When all the onlooker bees have been used, the most economical trajectory is memorized and allocated in memory. As the algorithm is able to give a solution from here, it could be stopped now, or it can be allowed to continue its calculation to refine the solution. This allows a full control of the execution time as it is not mandatory to wait for the end of the calculation to have a solution.

Phase IV - Scout bee phase: Regardless of the phase, every time a mutation fails to create a more economical solution, a counter associated to the studied trajectory is incremented. If a trajectory mutation succeeds, its counter is reseated. However, when the counter reaches a pre-defined number, the associated trajectory is discarded. This allows avoiding spending too much time on a trajectory that is not promising after many mutations, since it might be already the global optimal, or it is stuck in a local optimal. It is not important if the trajectory to be discarded the most economical so far, since it has already been memorized. The discarded trajectory is replaced by a new generic trajectory; based on the initialization process. This is what it is called a “scout bee”.If the maximal number of iterations is reached, the algorithm stops here. Otherwise, the algorithm returns to the Employed bee phase, and repeat all this process.

III. RESULTS

The trajectories shown below represent a flight from Montreal – Paris. The geodesic distance for this trajectory is of around 2970 nm. 36 waypoints were considered taking into account weather information. Mach number and flight level were kept constant along the flight. Two flight cases were examined.

Firstly, two schema of the calculated optimal trajectory are presented below. The interest is to show how the form of the trajectories influences the final result.

Fig 3 represents the first flight. The black lines represent the original flight plan (in the center) and the boundaries. The optimum flight plan found is schematized in red. All the trajectories owned by “employed bees” can be observed in green**.**

It is important to observe as well the comportment of the non-optimal trajectories. If it is supposed that all the research space has been covered by the created generic trajectories, then

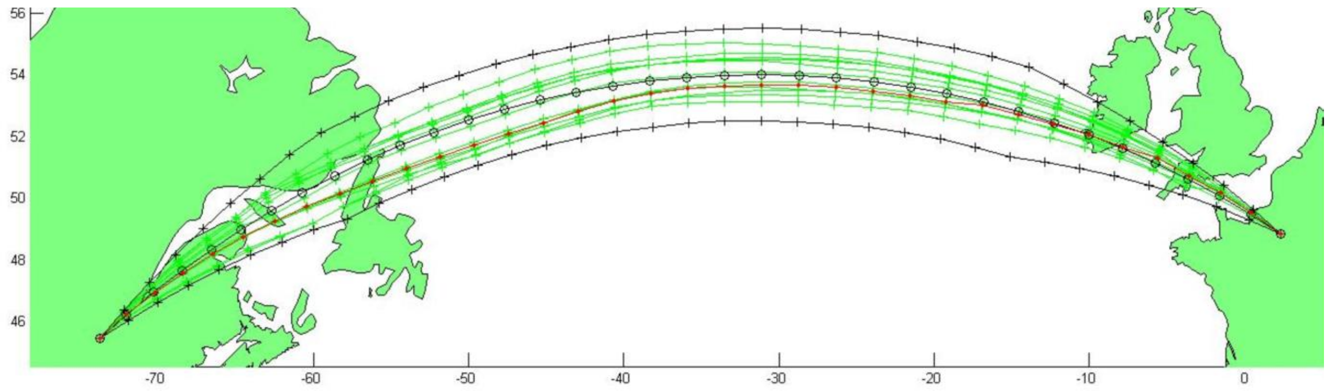

Fig 3. Flight Test 1: Optimized trajectory (red), and explored trajectories (green)

the empty space is the inefficient space. And so if many of them converge to the optimum trajectory, it suggests a strong solution near the optimal found.

In the example of Fig. 3, six trajectories out of twelve are close to the optimal. The other trajectories are searching the rest of the search space, and are most probably either trapped in a local maximum, or not enough developed. This is especially true when the employed bee has abandoned its old solution for a new generic one. However this comportment assures a better coverage of the area, and allows avoiding the local maximum efficiently.

The algorithm was executed again for the same trajectory using the same parameters. As the algorithm depends on the initial semi-random trajectories, and mutations are influenced by these trajectories, the same result is not expected as it can be seen in Fig 4. Although the most economical trajectory seems smoother than the one in Fig 3. the fuel consumption is higher for about 40kg. Looking at the green trajectories, it can be seen that the algorithm is not yet converging to the optimal solution.

The algorithm was executed 1200 times for a flight from Montreal to Paris, the 23th of June 2013. The speed of the plane was 0.8 Mach and its altitude was 33 000 feet. The shortest path in distance for this trajectory was around 2970 NM. The flight began at 12h00m00s 36 waypoints were considered, taking into account weather information.

Following the route of reference (geodesic) the plane requires 26,804 kg of fuel. For 1200 runs, the amount of fuel that the optimized trajectories were able to save was from 102kg to 164kg. The average amount of saved fuel is 140kg, and the median is 139kg. Fig.5 presents the fuel consumption for this test. The horizontal axis is the number of test result, from the lowest to the best, in term of saved fuel

Another variable that influences flight cost is flight time. The algorithm developed in this paper is as well able to save flight time. Following the geodesic reference trajectory, the aircraft required 22,288 s to complete its flight. The results provided by the optimized trajectories saved flight time in a range from 86 seconds to 142 seconds. The average saved time was 119 seconds, and the median was 119 seconds too. Results for the simulated flights can be seen in Fig 6 where the horizontal axis is the test number (sorted by fuel saved).

The algorithm computation required around 260 seconds to find the optimal trajectory. This is considered to be a good computation time result taking into account that 36 waypoints were considered. Computation time can be reduced even further at the stake of diminishing the quality of the solution found.

For every special case, the algorithm can be modified to find better result, as it was the case of a variant explored. In this variant the number of employed bees was two times higher, the iterations were reduced by half, and the mutation counter limit was changed. For the same flight, 240 test were executed, and it as observed that an increment of 10 kg of fuel was found. However, tailoring the algorithm per flight case is not practical as it is time consuming, and the search space can change on a daily basis.

During different algorithm tests, as expected, it was observed that weather has a strongly influence. By decreasing the aircraft Mach number to 0.62, the algorithm saved 186 kg of fuel, for a total consumption of 25,8 tons.

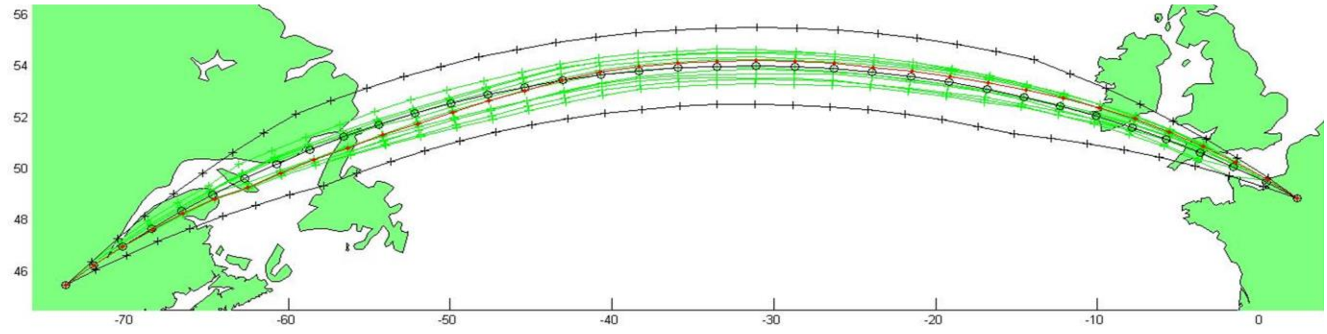

Fig 3. Flight Test 2: Optimized trajectory (red), and explored trajectories (green)

IV. CONCLUSION AND FUTURE WORK

An algorithm that optimized the direct trajectory for the lateral navigation trajectory was developed using weather data and an aircraft performance database. The developed ABC algorithm was relatively robust with a low computation time. The preliminary results showed that the optimized trajectories evaluated reduced the flight time, fuel burn, and thus flight cost.

More appropriate results can be found by changing the algorithm parameters such as the number of employed bee, the formula of rating during the onlooker bee phase, the maximum mutation fail allowed, and etc. A deeper study would be required to set the best combination of parameters.

If more time calculation was to be allowed, the ABC could create a smoother and more interesting optimal trajectory. The algorithm's ability to provide the optimal trajectory with a dynamic grid makes it theoretically able to be closer to the true optimal trajectory than an algorithm using a grid.

The low computation time required by the algorithm allows improving this algorithm. Future work consists in using the ABC to optimize the vertical navigation profile, and the speed schedule to fulfill the required time of arrival constraint.

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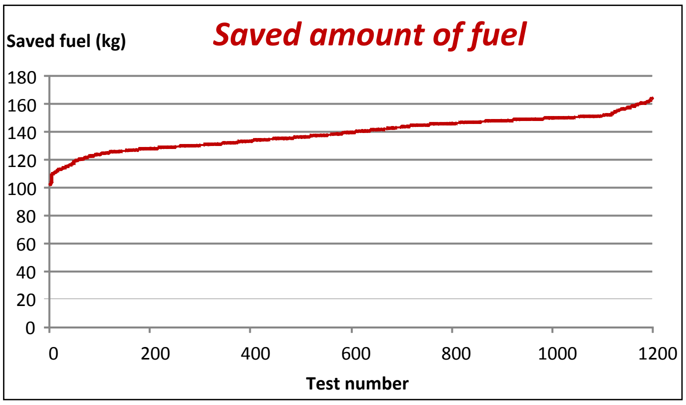

Fig. 5. Saved Fuel for 1200 trajectories

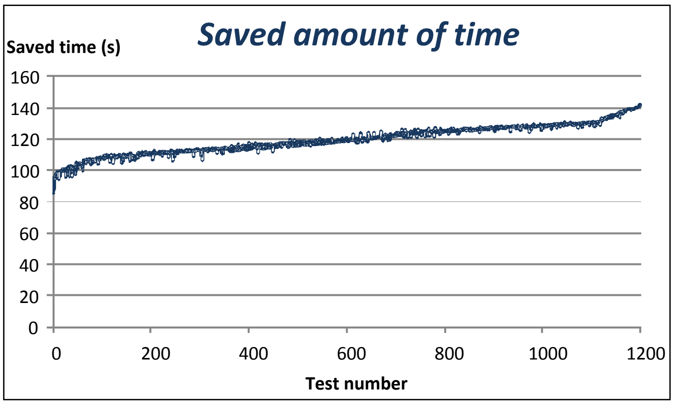

Fig 6 : Saved amount of time

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