

# Cessna Citation X Pitch Rate Control Design using Guardian Maps

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**Abstract**— Satisfying handling qualities remains one of the major concerns of flight control engineers. In addition to satisfy many stringent performances, Flight Control Systems (FCS) have to be robust to various uncertainties. Although modern control techniques can handle many types of constraints, fulfilling these requirements remains a challenge for engineers. It is therefore of interest to find a method that keeps the simplicity of classical architecture while taking advantage of modern techniques. In this paper, a new algorithm to design a pitch rate controller is presented. Based upon the guardian maps theory, the algorithm tries to find a controller that satisfies several performances expressed in terms of handling qualities. To validate the proposed methodology, simulations for 10 flight conditions have been performed using a full nonlinear level D aircraft model of the Cessna Citation X business aircraft. The results obtained showed that the proposed algorithm works very well.

**Keywords** — Cessna Citation X; flight control; robustness; guardian maps; handling qualities.

## I. INTRODUCTION

The beginning of the Fly-By-Wire technology early in the 1960s [1, 2] has led the aerospace industry to develop more evolved and efficient flight control systems in order to build safer and reliable airplanes. With the constant augmentation of aircraft in the sky, the need of designing flight control systems that are efficient and robust became one of the main goals of the aerospace industry. However, the development and the integration of flight control systems are costly and time-consuming. This is why in recent years, several researchers and engineers have focused their works to provide effective and robust controller design techniques .

The improvement of numerical optimization algorithms has greatly contributed to the development of modern control techniques such as  $H_\infty$  or  $\mu$ -synthesis [3, 4]. These two methods aim to find a controller that minimizes disturbances effects while stabilizing the system in closed-loop [4]. They are therefore helpful for the aerospace industry because they can fulfill many stringent constraints while remaining robust to parametric uncertainties.

Boughari *et al.* in [5] presented a procedure to design a robust controller for the Cessna Citation X aircraft business jet using the  $H_\infty$  theory. In this study, the  $H_\infty$  synthesis was combined with two meta-heuristic algorithms (the genetic algorithm and the differential evolution algorithm) in order to

find the optimal  $H_\infty$  weighting functions that describe the closed-loop performances. The methodology was applied to obtain a robust lateral controller for different flight conditions within the Cessna Citation X aircraft flight envelope. The controller was finally exported into Simulink and simulations using a full nonlinear aircraft model have proved the efficiency of the controller.

Similarly, Mystkowski in [6] presented a procedure based on the  $\mu$ -synthesis technique to design a robust longitudinal and lateral controller for a family of micro Unmanned Aerial Vehicles (UAV). The controller was first computed in Matlab using the robust control toolbox, and then optimized via fixed-point arithmetics. Finally, the controller was implemented in a single-board microcomputer in order to be tested on the real system. However, according to the author, the high order of the controller did not allow its implementation on any microcomputer.

To solve the implementation problem, Saussié *et al.* proposed in [7] to reduce the high order of the controller by using robust modal control techniques. In this study, the authors performed first a  $H_\infty$  synthesis in order to find aircraft pitch rate controller for the Bombardier Challenger 604 aircraft that satisfied several handling qualities while being robust to mass and center of gravity variations. The obtained controller was next reduced to make it similar to a classical structure usually used by the aerospace industry. Simulations for 8 aircraft flight conditions in terms of mass and center of gravity location were performed and promising results were obtained.

All these examples showed the improvement made with modern control techniques such as  $H_\infty$  or  $\mu$ -synthesis. However, even if the controllers designed with these techniques can handle many stringent constraints while being robust to uncertainties, their counterpart is their high order (at least the same order as the system). Aeronautical engineers still use the classical flight control approach mainly because the high order controller high order prevents its integration on the real system [8]. In addition to providing relatively simple controller architecture, classical methods allow a better understanding of the controller behavior.

Classical flight control systems are based on successive feedbacks and require a really good knowledge of the system [8, 9]. However, because of their simplicity, they cannot take into account uncertainties or stringent constraints as the

modern techniques do. It is therefore of interest to find a method that keeps the simplicity of a classical architecture while using modern techniques advantages.

Ghazi and Botez in [10, 11], proposed a simple fixed architecture controller for the Cessna Citation X business jet aircraft. As usually used in classical flight controls [12, 13], the controller architecture consisted of a Stability Augmentation System (SAS) and a Command Augmentation System (CAS). The gains for each loop (SAS and CAS) were computed with an automatic procedure based on a combination between the LQR theory [13-15], a genetic algorithm and the guardian maps approach [16]. The specific combination allowed the authors to obtain a controller for the whole aircraft flight envelope that was efficient and robust to various uncertainties. The controller was successfully applied on a Cessna Citation X full nonlinear model and very good results were obtained.

The procedure proposed by Ghazi and Botez assumed that all the aircraft states are available for the flight control system. However, in some cases, sensors used to measure specific flight parameters are really expensive. Consequently, only a few flight parameters are measurable, and the LQR method cannot be applied.

Saussié *et al.* in [17] presented a robustness augmentation algorithm for a fixed aircraft pitch rate architecture controller. The proposed algorithm relied upon the guardian maps theory and was used to improve the robustness of an initial controller that satisfied pole confinement constraints. The procedure was applied to design the pitch rate controller of a Challenger 604 aircraft and obtained results were promising. However, according to the authors, although the general principle of the algorithm remained relatively simple, the update of the gains inside the algorithm remained the most difficult part.

In this paper, a methodology to design a longitudinal pitch rate control system for the business aircraft Cessna Citation X is presented. The proposed procedure is based upon a classical fixed architecture controller mixed with an optimization algorithm that allow to find the best gains of the longitudinal flight control system in order to achieve given performance. The methodology has been validated using a nonlinear aircraft model of the Cessna Citation X built in Matlab/Simulink using data from a level D aircraft research flight simulator designed and manufactured by CAE Inc. According to the Federal Aviation Administration (FAA, AC 120-40B), the level D is the highest certification level that can be delivered by certification authorities for the flight dynamic.



Fig. 1. Level D Cessna Citation X Flight Simulator

This paper is arranged as follows. In Section 2, the Cessna Citation X aircraft model is introduced, a brief description of the controller architecture is provided, and the handling qualities of interest are exposed. Section 3 briefly presents the guardian maps theory. Section 4 shows the procedure used to tune the controller gain in order to achieve all the desired performance. Section 5 deals with the results and the validation of the algorithm. Finally, conclusions and future work is provided.

## II. FLIGHT CONTROLLER PROBLEM

This section aims to describe the Cessna Citation X aircraft longitudinal control problem. First, a brief description of the aircraft open-loop model is given, followed by a presentation of the controller architecture. Then, a list of requirements (handling qualities) is enumerated.

### A. Cessna Citation X Open Loop Model

In this paper, the Cessna Citation X aircraft is modelled using a six degrees of freedom nonlinear model developed by Ghazi and Botez in [18, 19]. However, for design purpose, this paper considers only the aircraft longitudinal motion. Using trim and linearization routines, the aircraft equations of motion have been linearized for different flight conditions in terms of altitude, speed, gross weight and center gravity position. Then, as usually done in flight control systems [20, 21], the phugoid mode was truncated and only the short period mode was considered. The short period approximation models obtained by linearization were next compared and validated using linear models obtained with system identification techniques from flight tests [22, 23].

The actuators and sensors dynamics are modelled using two fourth order transfer functions with delay due to data processing. After reduction using modal truncation, the high order open-loop transfer function was reduced to an 8<sup>th</sup> state space model denoted as:

$$\begin{aligned} \Delta \dot{\mathbf{x}} &= \mathbf{A} \Delta \mathbf{x} + \mathbf{B} \Delta \delta_c \\ \Delta q &= \mathbf{C}_q \Delta \mathbf{x} \\ \Delta n_z &= \mathbf{C}_{nz} \Delta \mathbf{x} \end{aligned} \quad (1)$$

where  $\Delta \mathbf{x}$  represents the aircraft, actuators and sensors vector state,  $\Delta \delta_c$  is the elevator command position,  $\Delta q$  is the aircraft pitch rate and  $\Delta n_z$  is the aircraft normal acceleration.

### B. Controller Architecture

To track the pitch rate commands  $\Delta q_{ref}$ , the classical controller architecture shown in Fig. 2 is used.

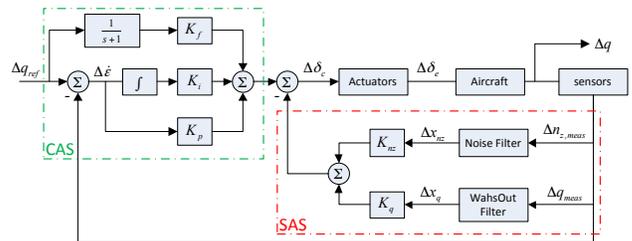


Fig. 2. Pitch Rate Controller Architecture

As usually done in classical flight controls [20, 21], the controller consists of a Stability Augmentation System (SAS) and a Command Augmentation System (CAS).

The SAS is composed of two feedback loops with fixed first order filters: a washout filter and a noise filter. The gains  $K_{nz}$  and  $K_q$  are adjustable gains and must be tuned in order to improve the aircraft stability. The CAS loop is formed of a proportional-integral controller and a feedforward loop. The first order transfer function of the feedforward loop has been added in order to make the command smoothest. The three gains  $K_p$ ,  $K_i$  and  $K_f$ , are also adjustable and must be designed in order to improve the aircraft handling qualities.

Finally, the performances of the closed-loop are governed by the set of gains  $\mathbf{K} = \{K_q, K_{nz}, K_p, K_i, K_f\}$ .

### C. Longitudinal Flight Requirements

The longitudinal flight requirements are the minimum acceptable standards to which the stability, control and handling of the aircraft must be designed. They are used to make sure that the aircraft has good flying and handling qualities. The flying qualities (FQs) concern how well a long-term task can be fulfilled, while handling qualities (HQs) represent how the aircraft behaves at short-term to specific inputs [21]. The considered boundaries of the flying and handling qualities according to military standards [24] are given in Table 1. They are expressed in terms of short period damping ratio  $\zeta_{sp}$ , settling time  $ST$ , steady state error  $SSE$ , Gibson dropback  $Db$ , gain margin  $M_G$  and phase margin  $M_\phi$ .

The Gibson dropback is a short-term measure of the pitch attitude. It is usually relevant when the pilot is trying to change the pitch rate ( $q$ ) of the aircraft. The dropback can be calculated based on the reduced-order attitude  $\theta$  response to a stick step. Examples of positive, zero and negative dropback are illustrated in Fig. 3.

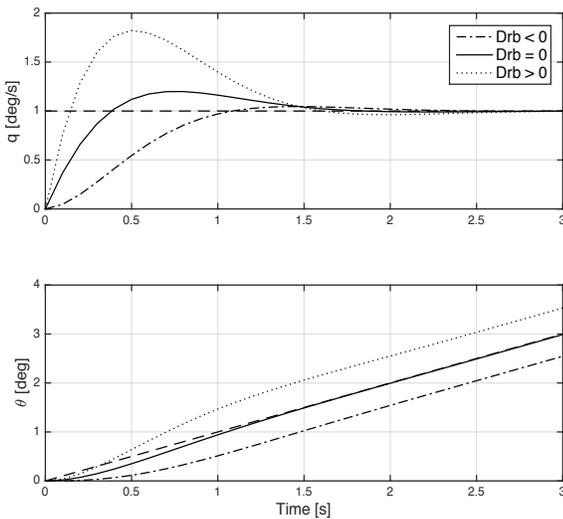


Fig. 3. Dropback illustration

TABLE I. FLYING AND HANDLING QUALITIES BOUNDARIES

HQs / FQs	Level 1	Good Level 1
$\zeta_{sp}$	$0.35 \leq \zeta_{sp} \leq 1.35$	$0.7 \leq \zeta_{sp} \leq 1.35$
$SSE$	$SSE \leq 0.1$ (deg/s)	$SSE \leq 0.1$ (deg/s)
$ST$	$ST(2\%) \leq 3$ (s)	$ST(1\%) \leq 3$ (s)
$Db$	$-0.2 \leq Db \leq 0.5$	$0.0 \leq Db \leq 0.3$
$M_G$	$M_G \geq 6$ dB	
$M_\phi$	$M_\phi \geq 45^\circ$	

### III. AN INTRODUCTION TO THE GUARDIAN MAPS

The guardian map approach was introduced by Saydy *et al.* [16] as a tool for the study of “generalized stability” of parameterized families of matrices (or polynomials). The “generalized stability” refers to the confinement of matrix eigenvalues to general open subsets of the complex plan.

#### A. Definition

Basically, guardian maps are scalar functions defined for a specific system that vanish whenever the system is at the limit of stability. The system of interest can be represented either by a set of  $n \times n$  real matrices or by an  $n^{\text{th}}$ -order polynomials. To simplify the study, the definitions that follow are directly applied to families of matrices. However, it can be easily adapted to polynomials.

**Definition 1:** Let  $\Omega$  be an open subset of the complex plan of interest. The system defined by the Eq. (1) is stable relative to  $\Omega$  if the matrix  $\mathbf{A}$  have all its eigenvalue in  $\Omega$ , i.e. if  $\sigma(\mathbf{A}) \subset \Omega$ .

Here  $\sigma(\mathbf{A})$  denotes the set consisting of all the eigenvalue of  $\mathbf{A}$ . Thus, the set of all matrices, which are stable relative to  $\Omega$ , can be therefore defined such as:

$$S(\Omega) = \{\mathbf{A} \in \mathbb{R}^{n \times n} : \sigma(\mathbf{A}) \subset \Omega\} \quad (2)$$

Based on this last definition, the mathematical formulation of the guardian maps can be defined as follow [16]:

**Definition 2:** Let  $v$  map  $\mathbb{R}^{n \times n}$  into  $\mathbb{C}$ . We say that  $v$  guards  $S(\Omega)$  if for all  $\mathbf{A} \in \bar{S}(\Omega)$ , the following equivalence holds:

$$v(\mathbf{A}) = 0 \Leftrightarrow \mathbf{A} \in \partial S(\Omega) \quad (3)$$

Here  $\bar{S}$  denotes the closure of the set  $S$  and  $\partial S$  its boundary. We say that  $v$  is a guardian map for  $S$ .

#### B. Guardian Maps Examples

To illustrate the concept of guardian maps, let’s consider the three most classical stability regions of the complex plan illustrated in Fig. 4.

**Negativity Margin:** the open  $\alpha$ -shifted left half-plane region defined by  $\{\lambda \in \mathbb{C} : \text{Re}(\lambda) < \alpha\}$  is guarded by:

$$v_\alpha(\mathbf{A}) = \det(\mathbf{A} \odot \mathbf{I} - \alpha \mathbf{I} \odot \mathbf{I}) \det(\mathbf{A} - \alpha \mathbf{I}) \quad (4)$$

where  $\odot$  denotes the bialternate product of two matrices.

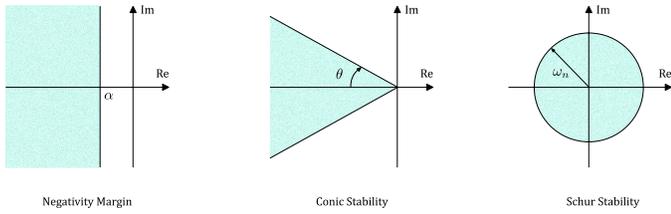


Fig. 4. Classical stability regions of the complex plan

**Damping Stability:** the region delimited by the damping cone with the half-angle  $\theta = \text{acos}(\zeta)$  is guarded by:

$$v_{\zeta}(\mathbf{A}) = \det(\mathbf{A}^2 \odot \mathbf{I} + (1 - 2\zeta^2)\mathbf{A} \odot \mathbf{A}) \det(\mathbf{A}) \quad (5)$$

where  $\zeta$  is the limiting damping ratio.

**Schur stability:** the region defined by the open disk with a radius  $\omega_n$  is guarded by:

$$v_{\omega_n}(\mathbf{A}) = \det(\mathbf{A} \odot \mathbf{A} - \omega_n^2 \mathbf{I} \odot \mathbf{I}) \det(\mathbf{A} - \omega_n \mathbf{I}) \det(\mathbf{A} + \omega_n \mathbf{I}) \quad (6)$$

It can be noticed from Eq. (4) that the Hurwitz stability (i.e. the open left half-plan) is a simple case of the negativity margin with  $\alpha = 0$ . However, a systematic method of constructing guardian maps for other regions that those considered in this paper can be found in [16].

### C. Two-parameters family matrices stability test

Let  $(S_r)$  be a two-parameters family of linear systems described by the following general form:

$$(S_r) \equiv \begin{cases} \Delta \dot{\mathbf{x}} = \mathbf{A}(\mathbf{r})\Delta \mathbf{x} + \mathbf{B}(\mathbf{r})\Delta \boldsymbol{\eta} \\ \Delta \mathbf{y} = \mathbf{C}(\mathbf{r})\Delta \mathbf{x} + \mathbf{D}(\mathbf{r})\Delta \boldsymbol{\eta} \end{cases} \quad (7)$$

where  $\mathbf{r} \in \mathbb{R}^2$  is a parameter vector where each parameter  $r_i$ ,  $i = \{1,2\}$  lies in a given range for which only the bounds are known, say  $\mathbf{r} \in U \subset \mathbb{R}^2$  (i.e.  $r_i \in [\underline{r}_i, \bar{r}_i], \forall i \in \llbracket 1,2 \rrbracket$ ).

To test if the two-parameters family  $(S_r)$  is stable relative to an open subset of the complex plan  $\Omega$  for all  $\mathbf{r} \in U$ , the following theorem and corollary can be used.

**Theorem 1:** (Saydy *et al.* [16]) *Let  $S(\Omega)$  be guarded by the map  $v_{\Omega}$ . The family  $\{\mathbf{A}(\mathbf{r}) : \mathbf{r} \in U\}$  is stable relative to  $\Omega$  if and only if:*

- (i). *It is nominally stable, i.e.  $\mathbf{A}(\mathbf{r}_0) \in S(\Omega)$  for some  $\mathbf{r}_0 \in U$ ; and,*
- (ii).  *$v_{\Omega}(\mathbf{A}(\mathbf{r})) \neq 0$ , for all  $\mathbf{r} \in U$ .*

**Corollary 1:** *Let  $S(\Omega)$  be guarded by the map  $v_{\Omega}$  and consider the family  $\{\mathbf{A}(\mathbf{r}) : \mathbf{r} \in U\}$ . Then the set  $C$  defined by:*

$$C = \left\{ \mathbf{r} \in \mathbb{R}^k : v_{\Omega}(\mathbf{A}(\mathbf{r})) = 0 \right\} \quad (8)$$

*divides the space parameters  $\mathbb{R}^k$  into components  $C_i$  that are either stable or unstable relative to  $\Omega$ . Then, to see which situation prevails for a given component  $C_i$ , one simply has to test  $\mathbf{A}(\mathbf{r})$  for any one vector in  $C_i$ .*

## IV. CONTROLLER DESIGN METHOD

This section introduces the algorithm based upon the guardian maps. First, the procedure is applied to a specific case in order to better illustrate the main steps of the algorithm. Subsequently, a procedure for solving the flight controller problem described in section **Flight Controller Problem** is presented.

### A. Two Degrees of Freedom Controller Design Example

To illustrate the proposed algorithm, we consider here the synthesis of a PI controller for the unstable system illustrated in Fig. 5,

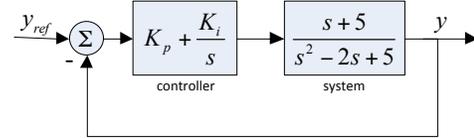


Fig. 5. PI Feedback Configuration

where the equivalent state-space representation of the closed-loop is given by the following equation:

$$\mathbf{A}(K_p, K_i) = \begin{bmatrix} 2 - K_p & -5K_p - 5 & K_i \\ 1 & 0 & 0 \\ -1 & -5 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} K_p \\ 0 \\ 1 \end{bmatrix} \quad (9)$$

In addition to stabilise the initial system, the PI controller must place the pole of the closed-loop inside a sub-region of the complex plan defined by  $\Omega_t(-1.5, 0.7, 12) = \{\lambda \in \mathbb{C} : \text{Re}(\lambda) < -1.5, \zeta(\lambda) > 0.7, |\lambda| < 12\}$ .

As shown in Fig. 6, the algorithm and the synthesis procedure consist essentially of four steps:

- **Step 1:** using Eqs. (4)-(6), the algorithm computes the guardian maps of  $\mathbf{A}(K_p, K_i)$  for  $\Omega_t$ , and finds the contours of the map that reveal for which combination of  $(K_p, K_i)$  the guardian maps vanish (see Fig. 6 – step 1). According to **Definition 2**, these contours reveal the values of  $(K_p, K_i)$  that bring the closed-loop at the limit of the sub-region  $\Omega_t$ .
- **Step 2:** the algorithm researches a region in which the closed-loop is stable relative to  $\Omega_t$ . To do that, the algorithm selects randomly five points along each contour and verifies if the neighbourhood of each point ensure that all the eigenvalues of  $\mathbf{A}(K_p, K_i)$  are inside  $\Omega_t$  (see Fig. 6 – step 2). If such a point exists, it is then selected as starting point.
- **Step 3:** the algorithm builds a simplex by selecting randomly three points in the neighbourhood of the starting point (see Fig. 6 – step 3).

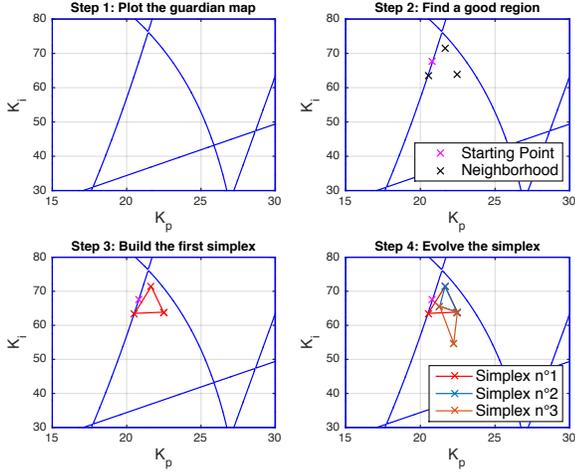


Fig. 6. Search Algorithm Illustration

- **Step 4:** once the starting point and the first simplex have been identified, the main intuitive idea is to move sufficiently far from the contour, hence, the resulting closed-loop poles will also move well inside the sub-region  $\Omega_t$ . To do that, the algorithm uses different geometric transformations to move the simplex to the center of the region. These transformations are mainly based on the optimization algorithm proposed by Nelder-Mead in [25]. At each iteration, the algorithm selects the vertex of the simplex that is the closest from the boundary and performs a reflection with respect to the other two vertices. If the reflection is not possible, then the algorithm tries a contraction inside the simplex. The algorithm stops when all the vertices of the simplex are close to each other, which means that the simplex cannot evolve anymore.

Table 2 and Figure 7 show the results obtained after 47 iterations.

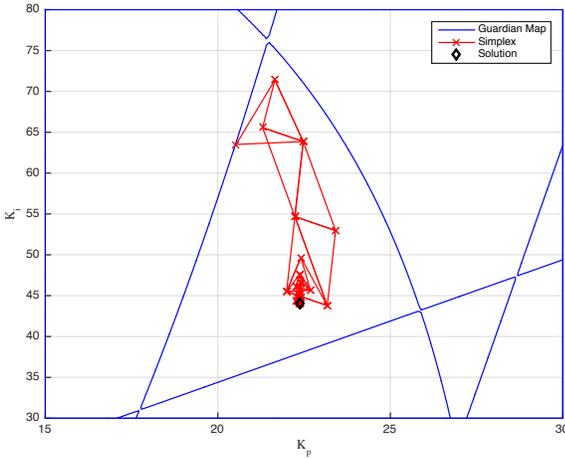


Fig. 7. Algorithm Illustration

TABLE II. CLOSED-LOOP POLES

Loop	Poles ( $\lambda$ )	$Re(\lambda)$	$\zeta(\lambda)$	$ \lambda $
Open-Loop	$1.0 \pm 2.0i$	1.0	-0.447	2.24
Closed-Loop	$-1.71$ $-9.33 \pm 6.48i$	-1.71 -9.38	1.0 0.821	1.71 11.4

As it can be observed, the results are very good. In addition to stabilize the system, the algorithm finds the “center” of the sub region where the closed-loop is stable relative to  $\Omega_t$ .

### B. Design Procedure for the Cessna Citation X

The main goal of the algorithm is to tune the set of gains  $\mathbf{K} = \{K_q, K_{nz}, K_p, K_i, K_f\}$  in order to place the closed-loop poles inside a specific sub-region  $\Omega_t$  of the complex plan that represents the required closed-loop performances in terms of damping ratio and settling time. As shown in the previous section, the proposed algorithm can deal only with two parameters. Therefore, the SAS and the CAS have to be designed one at a time.

The synthesis procedure can be summarized as:

1. **Design of the SAS ( $K_p = K_i = K_f = 0$ ):** using the procedure described in the previous section, the gains  $K_q$  and  $K_{nz}$  are computed with  $\Omega_t(-1.0, 0.5, \infty)$ .
2. **Design of the CAS ( $K_f = 0$ ):** using the procedure described in the previous section and the results obtained in step 1, the gains  $K_p$  and  $K_i$  are computed with  $\Omega_t(-0.5, 0.6, \infty)$ .
3. Finally, according to Saussié *et al.* in [17], the gain  $K_f$  is chosen to set zero the dropback by solving the following equation:

$$0 = \frac{C_{cl} A_{cl}(K_f)^{-2} B_{cl}(K_f)}{C_{cl} A_{cl}(K_f)^{-1} B_{cl}(K_f)} \quad (10)$$

where  $A_{cl}$ ,  $B_{cl}$  and  $C_{cl}$  are the state matrix of the closed-loop in Fig. 2.

## V. SIMULATION AND RESULTS

The algorithm was applied to 10 flight conditions in terms of altitude, speed, gross weight and center of gravity position. These flight conditions were selected within the Cessna Citation X flight envelope.

Figures 8 and 9 show the aircraft nonlinear model time response and the Bode diagram for all the 10 flight conditions. As it can be observed, the aircraft is successfully controlled. The settling time for all the ten models is less than 2 seconds and the steady state error is also less than 0.1 deg/s. Regarding the dropback it remains well below 0.3 as imposed by the performance in Table 1.

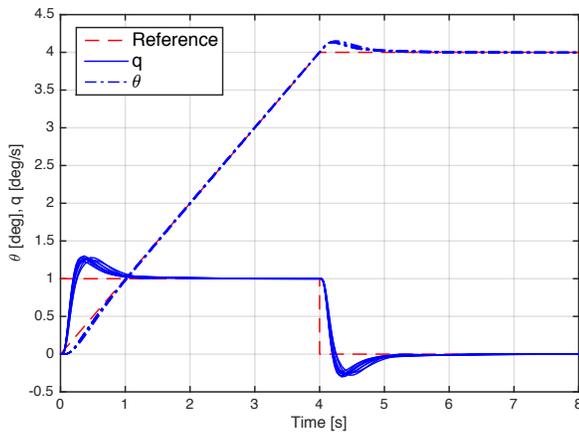


Fig. 8. Aircraft Time Response for 10 Flight Conditions

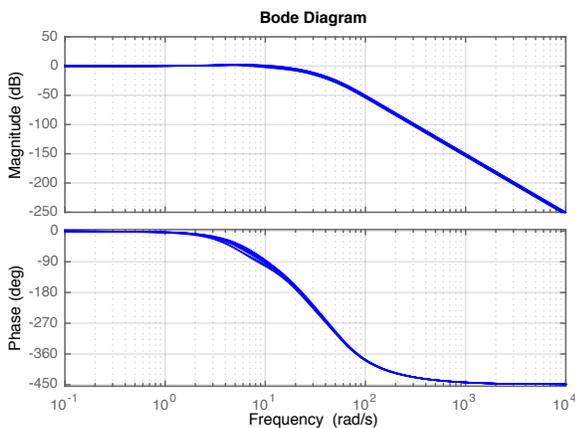


Fig. 9. Aircraft Bode Diagram for 10 Flight Conditions

Finally, as shown in Fig. 9, the gain margin and the phase margin are above the minimum values imposed by the handling qualities in Table 1.

#### CONCLUSION

In this paper, a new design procedure to obtain a controller to track the pitch rate command was presented. Based upon the guardian maps theory, the algorithm found the gains for a controller that satisfies several performances expressed in terms of handling qualities. The controller was successfully applied on a Cessna Citation X business aircraft nonlinear model, and very good results were obtained. To improve the algorithm, the next steps are suggested:

- The actual algorithm cannot change the space of research. If the algorithm did not find a good starting point, the algorithm stops and concludes that there are no possible solutions. To improve this part of the algorithm, a new function should be added to allow the algorithm to decide if the search area should be increased or not in order to find a better solution.

- The methodology used to select the starting point should also be improved in order to reduce the algorithm execution time.

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